Flexible phase synchronisation control method using partially unlocking oscillator arrays

H.-A. Tanaka, K. Shimizu, O. Masugata and T. Endo

A phase synchronisation method is proposed for beam-scanning control, utilising a newly identified phase synchronisation pattern. The proposed method provides more flexible control compared to conventional methods, even in noisy, non-ideal environments, as confirmed by systematic simulations and mathematical analysis.

Introduction: Coupled oscillator arrays emerge in a wide range of engineering issues. Examples include millimetre-wave power-combining and beam-scanning control systems, central pattern generators (CPG) in robotics, and Josephson junction arrays. In contrast to these examples that utilise the mutually locked (synchronised) state, little attention has been paid to unlocking states, presumably because few applications have been sought in such unlocking situations. In beam-scanning control systems using coupled oscillator arrays, a linear phase progression must be maintained across the array for beam forming (see Fig. 1), and the radiated beam is steered by controlling the phase difference between the adjacent oscillators. Methods of controlling such phase differences have been proposed and demonstrated, respectively, by Stephan [1] and by Liao and York [2], where oscillators at both ends of the array (oscillators 1 and N in Fig. 1*a*) are controlled to steer the beam. In [2], antisymmetrical frequency detuning is applied to the oscillators at the ends of the array. In such coupled oscillator-based beam steering, the coupling is loose (weak) when the oscillators are coupled radiatively for instance. Accordingly, as shown in [2] when a particular scan angle, say $+12.5°$, is required, the frequencies of the end oscillators must be 9.985 and 10.015 GHz, respectively, for instance, while the other oscillators have a frequency of 10.0 GHz. This suggests the end oscillators require careful frequency control since the frequency control resolution for the end oscillators must be within the order of a few kilohertz.

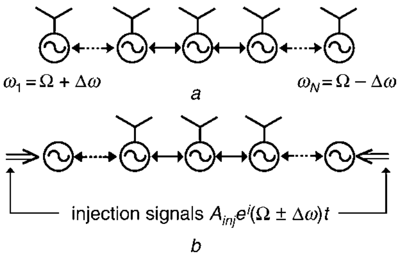

Fig. 1 *Coupled oscillator arrays for beam-scanning*

a Conventional system
b Proposed, partially unlocking system

In this Letter, we consider a counterpart of the injection-locked state, i.e. unlocking states, which emerge quite naturally. In the above example, if the end oscillators have frequencies of approximately 9.0 and 11.0 GHz, respectively, for instance, then they are unlocking with respect to the other oscillators. We have analysed such unlocking states, and found that a robust, exactly linear phase progression is still obtained in the array, except for a few oscillators near both ends. In the present study, this somewhat counter-intuitive phenomena is systematically analysed both numerically and theoretically, leading to a closed formula of the phase progression for given frequencies and amplitudes of the end oscillators (or injection signals). Based on this robust phase progression in the unlocking state, a method of beam-scanning control is proposed, which does not require high frequency control resolution.

Phase synchronisation in injection-locked oscillator arrays: For weakly coupled quasi-optical oscillators, York [3] developed a systematic reduction of model equations. In this Section, we follow this reduction and explain how the linear phase progression is realised in the coupled oscillator array. Contrary to previous studies, we consider herein an injection-unlocking state, and analyse the phase relationship in the oscillator array. We also assume a weak coupling between adjacent oscillators, as well as sufficiently uniform oscillator characteristics. Under such conditions, a systematic derivation of the phase equation for oscillators can be constructed [3], which eventually takes the following form:

$$\dot{\theta}_i = \omega_i + \kappa \sum_j \frac{A_j}{A_i} \sin(\Phi + \theta_j - \theta_i) \qquad (1)$$

where θ_i, A_i and ω_i represent the oscillation phase, amplitude and free-running frequency of the ith oscillator, respectively. Based on the above assumptions, $A_{i,j} \sim 1$ holds and $\kappa A_j/A_i \equiv \kappa$ is denoted by $\Delta\omega_m$. This $\Delta\omega_m$ is interpreted as the locking range of each oscillator, which is assumed to be small as well as identical for all oscillators. The phase lag Φ reflects the signal delay, which cannot be neglected for the case of radiative coupling. However, if the coupling is constructed from one-wavelength waveguides, Φ is assumed to be 0, and we focus on this case. Similar to Liao and York [2], we consider herein the frequency distribution as: $\omega_1 = \Omega + \Delta\omega$, $\omega_N = \Omega - \Delta\omega$, $\omega_2, \ldots, \omega_{N-1} = \Omega$, where Ω and $\Delta\omega$ denote the common locked frequency and the frequency detuning, respectively. Hereinafter, $\Delta\omega_m$ is set to 1 without loss of generality, and all numerical integrations are carried out by the fourth-order Runge-Kutta method with a step size of 0.001.

In contrast to the locked state of linear phase progression, we consider an unlocking state, which is obtained when $|\Delta\omega| > \Delta\omega_m$. We systematically changed $\Delta\omega/\Delta\omega_m$ from 1.0 to 20.0 and observed the phase relationships. Fig. 2 shows a typical unlocking phase relationship numerically observed in (1), where $\Delta\omega/\Delta\omega_m$ is set to 5.0, and a snapshot of $\theta_{i+1} - \theta_i$ is taken at $t = 5000$. This example exhibits the following three characteristics observed in these unlocking cases:

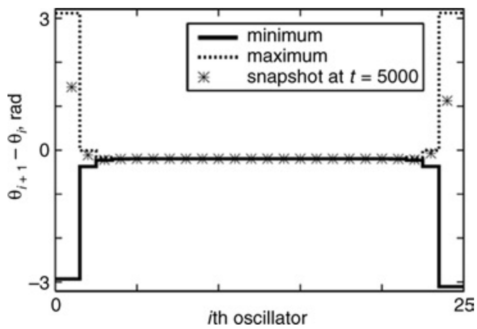

Fig. 2 *Phase progression pattern observed in unlocking system (1)*

(i) Oscillators near both ends of the array oscillate at common frequencies close to $\Omega \pm \Delta\omega$, respectively, and the other oscillators oscillate at Ω.
(ii) Inside the array, a linear phase progression appears, in which the phase difference $\theta_{i+1} - \theta_i$ $(i = 4, \ldots, 21)$ becomes time-constant and its fluctuation is always negligibly small. In contrast to the inside of the array, the oscillators near both ends exhibit a certain amount of fluctuation in phase difference $\theta_{i+1} - \theta_i$. The ranges for these fluctuations are shown by as the 'minimum and maximum phase differences' in Fig. 2.
(iii) In such a phase progression, the phase difference $\phi_i - \phi_{i+1} (\equiv \Delta\phi)$ is measured as $\Delta\phi \sim A_{inj}^2/\Delta\omega$ if the end oscillators have the same oscillation amplitude $A_1 = A_N \equiv A_{inj}$.

Phase control by unlocking oscillators: Based on these characteristics, an application of the unlocking array is suggested to control the linear phase progression both by the amplitude A_{inj} and the detuning $\Delta\omega$ at the end oscillators. To consider these observations theoretically, we introduce a slightly modified version of (1):

$$\begin{aligned}
\dot{\theta}_1 &= \Omega + A_{inj}\sin((\Omega + \Delta\omega)t - \theta_1) + \sin(\theta_2 - \theta_1) \\
\dot{\theta}_i &= \Omega + \sin(\theta_{i+1} - \theta_i) + \\
&\quad \sin(\theta_{i-1} - \theta_i), \quad (2 \le i \le N-1) \\
\dot{\theta}_N &= \Omega + A_{inj}\sin((\Omega - \Delta\omega)t - \theta_N) + \\
&\quad \sin(\theta_{N-1} - \theta_N)
\end{aligned} \qquad (2)$$

where the end unlocking oscillators in (1) are replaced by the external injection signals $A_{inj}^{1,N} e^{i(\Omega \pm \Delta\omega)t}$. This modification is shown schematically in Fig. 1*b*. Thus, the modification is not essential to the phase relationship considered here, and by this modification the analysis of (2) becomes much more tractable, as follows. First, a new variable $\phi_i \equiv \theta_i - \Omega t$ is introduced to (2), which yields:

$$\dot{\phi}_1 = A_{inj}^1 \sin(\Delta\omega t - \phi_1) + \sin(\phi_2 - \phi_1),$$
$$\dot{\phi}_N = A_{inj}^N \sin(-\Delta\omega t - \phi_N) + \sin(\phi_{N-1} - \phi_N)$$
$$\dot{\phi}_i = \sin(\phi_{i+1} - \phi_i) + \sin(\phi_{i-1} - \phi_i), \quad (2 \le i \le N-1) \quad (3)$$

For the long-term, slow movement of the phase relationship $\phi_i - \phi_{i+1}$, we reduce (3) by averaging the fast moving terms $A_{inj}\sin(\Delta\omega t - \phi_1)$ and $A_{inj}\sin(-\Delta\omega t - \phi_N)$, assuming $\Delta\omega t$ as the fast variable. In this averaging, a somewhat technical calculation is possible, using a nonlinear transformation of variables. This result is mathematically validated for large $\Delta\omega$ limit. Owing to lack of space, we omitted details. More general results will be reported elsewhere. Finally, (3) is averaged to yield:

$$\dot{\phi}_1 = \frac{(A_{inj}^1)^2}{2\Delta\omega} + \sin(\phi_2 - \phi_1)$$
$$\dot{\phi}_N = -\frac{(A_{inj}^N)^2}{2\Delta\omega} + \sin(\phi_{N-1} - \phi_N),$$
$$\dot{\phi}_i = \sin(\phi_{i+1} - \phi_i) + \sin(\phi_{i-1} - \phi_i) \quad (4)$$

Interestingly, (4) takes the form of (1), and the phase progression $\Delta\phi \equiv \phi_i - \phi_{i+1}$ is explicitly given as

$$\Delta\phi = \sin^{-1}\left(\frac{A_{inj}^2}{2\Delta\omega}\right) \text{ where } A_{inj} \equiv A_{inj}^1 = A_{inj}^N \quad (5)$$

which explains the observed phase progression inside the array.

Simulation results: Based on the closed formula (5) of the phase progression, a new beam-scanning control method is proposed. First, we check the consistency of the simulation results from (3), (4) and the closed formula of $\Delta\phi$ (5). Fig. 3 shows typical examples of these three data obtained for $\Delta\omega = 5.0$ and 10.0 (in a normalised frequency), where they are verified to be in good agreement. Also, from systematic simulations, it is observed that the consistency becomes better as $\Delta\omega$ increases. However, when $\Delta\omega$ becomes small, say, 0.1, the consistency decreases because $\dot{\phi}_1 \sim \Delta\omega$ (or $\dot{\phi}_N \sim -\Delta\omega$) is no longer large enough for validating the averaging results. Thus, the numerical simulations and the analytical results (4) and (5) suggest that the proposed method stably controls the linear phase progression by tuning A_{inj} and/or $\Delta\omega$.

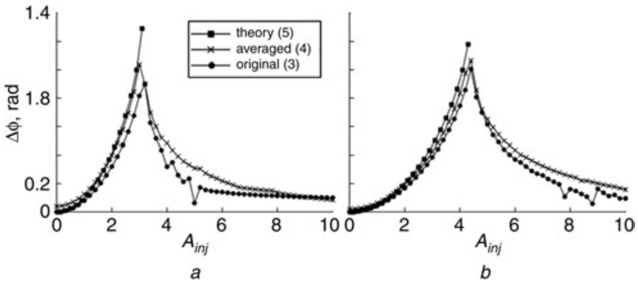

Fig. 3 *Comparison of phase progression Δφ obtained by (3), (4) and (5)*
a $\Delta\omega = 5.0$
b $\Delta\omega = 10.0$

The issues of robustness in the proposed method, i.e. noise immunity and the effect of asymmetric detuning frequencies, are considered. Extensive numerical simulations using random initial conditions are carried out according to [4], assuming the same white noise in both cases. Results suggest that the method of Liao and York [2] and the proposed method have approximately the same noise immunity under the above condition.

The effect of asymmetric detuning frequencies has also been considered numerically, both for the method of [2] (1) and for the proposed method (4). Results show that the amounts of distortion in the linear phase progression are approximately the same for both methods (data not shown due to lack of space). This suggests that both methods have approximately the same robustness as the asymmetric detuning frequencies. However, the proposed method effectively controls the phase progression by tuning the injection amplitude. In the above example using asymmetric detuning δ, the proposed method tunes the amplitudes A_{inj}^1 and/or A_{inj}^N according to (5), resulting in perfect linear phase progression.

Conclusions: We have proposed a novel control method for a linear phase pattern in coupled oscillator arrays, which provides more flexible and possibly robust control ability in coupled oscillators with small locking ranges. The phase pattern is clearly described in (5) and its validity and usefulness is confirmed by systematic simulations.

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References

1 Stephan, K.D.: 'Inter-injection-locked oscillators for power combining and phased arrays', *IEEE Trans. Microw. Theory Tech.*, 1986, **MTT-34**, pp. 1017–1025
2 Liao, P., and York, R.A.: 'A new phase-shifterless beam-scanning technique using arrays of coupled oscillators', *IEEE Trans. Microw. Theory Tech.*, 1993, **MTT-41**, pp. 1810–1815
3 York, R.A.: 'Nonlinear analysis of phase relationships in quasi-optical oscillator arrays', *IEEE Trans.*, 1993, **MTT-41**, pp. 1799–1809
4 Chang, H.-C., *et al.*: 'Phase noise in coupled oscillators: theory and experiment', *IEEE Trans.*, 1993, **MTT-41**, pp. 1799–1809